SINGLE WAVES IN UNDERGROUND HYDROMECHANICS PROBLEMS

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Models of single filtration waves, which can be used in problems of intensification of the development and investigation of oil–gas strata, are considered.

1. Hydrodynamic Waves. A structurally nonequilibrium relaxation fluid is considered whose filtration velocity due to the pressure gradient (P_x) is described by the relationship

$$\mathbf{W} = -\lambda\{z^n - \tau[(z^n)_t - (z^n)_t^0]\}\mathbf{P}_x/z; \tag{1.1}$$
$$z = |P_x|,\ \tau = \nu/\lambda z,\ n > 1,\ \lambda = \text{const} > 0$$

(λ, ν are the mobility and nonequilibrium coefficients).

The equilibrium flow of the system $\mathbf{W} = \mathbf{W}_0 = -\lambda z^n \mathbf{P}_x/z$ is pseudoplastic; in the nonequilibrium state the fluid opposes the change in flow velocity W_0 in time if the vectors $\boldsymbol{\nu}$ and $\mathbf{P}_x$ are codirectional ($\nu > 0$), and accelerates the flow in case the vectors $\boldsymbol{\nu}$ and $\mathbf{P}_x$ ($\nu < 0$) are opposite in direction, independently of whether the pressure gradient increases of decreases in proportion to its rate of change $(z^n)_t$.

Flows are considered that satisfy $W \to W_0$ as $z \to z_0 = \text{const}(t)$ with the asymptotic $(z^n)_t$ T $(z^n)_t^0 = z^\alpha \text{const}(\alpha \geqslant 1)$ which is not certainly zero for $z_0 \neq 0$, and with the characteristic relaxation time for the nonequilibrium effects τ at each point of the system, inversely proportional to the pressure gradient or Darcy flow velocity (λz) at this same point.

The continuity equation for the elastic filtration mode of a slightly compressible fluid in an elastically deformable porous medium (e.g., [1])

$$(m\rho)_t = -(\rho W)_x \approx \rho_0 (W)_x;\ \rho = \rho_0 \beta P, \tag{1.2}$$
$$m,\ \rho_0,\ \beta = \text{const} > 0$$

results for $\alpha = 1$ in the relationship

$$P_t = \varkappa (z^n)_x - \mu\,[(z^n)_t/z]_x;\ \varkappa = \lambda/m\beta,\ \mu = \nu/m\beta, \tag{1.3}$$

which is invariant relative to the coordinate transformation $(x;\ t) \Longleftrightarrow (-x;\ t)$ (by assumption the direction of the vectors $\boldsymbol{\mu}$ and $\vec{\partial}/\partial x$ is interrelated).

Here P and ρ are understood to be the differences between the running and initial pressures and the fluid densities corresponding to these pressures.

Let us define (1.3) in the form of stationary single waves $P(\xi)$, $\xi = x + Ut$ in a system ($\partial/\partial x = \partial/\partial\xi$, $\partial/\partial t = U\partial/\partial\xi$) moving with the wave velocity U

$$-\mu U[(z^n)_\xi/z]_\xi + \varkappa(z^n)_\xi - kUP_\xi = 0. \tag{1.4}$$

(The coefficient k is introduced for covenience of subsequent analogies in Sec. 2 with temperature waves, but $k \equiv 1$ in Sec. 1).

After a single integration of (1.4) with respect to ξ with $\mu(z^n)_\xi/z = \mu(z^n)_P$, $\boldsymbol{\mu}\cdot\mathbf{P}_\xi = \mu z > 0$, $\mu > 0$ taken into account for an inertial counteraction and $\boldsymbol{\mu}\cdot\mathbf{P}_\xi = \mu < 0$, $\mu < 0$ for acceleration, we have

$$-\mu U(z^n)_P + \varkappa z^n - kUP = S \doteq \text{const}. \tag{1.5}$$

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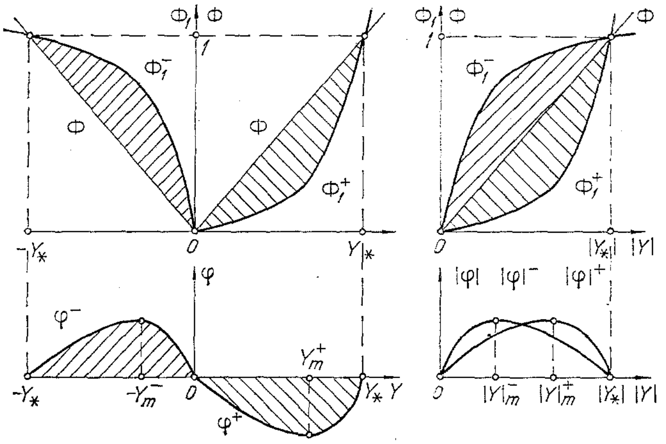

Fig. 1. Form of the dependences $\Phi(Y)$, $\Phi_1(Y)$, $\varphi(Y)$, $\Phi(|Y|)$, $\Phi_1(|Y|)$ and $\varphi(|Y|)$.

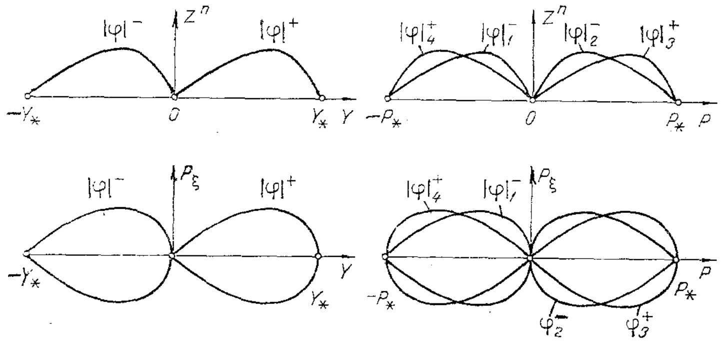

Fig. 2. Phase trajectories $z^n(Y)$, $P_\xi(Y)$, $z^n(P)$ and $P_\xi(P)$ for waves of the type 1, 2, 3, 4 according to (1.6).

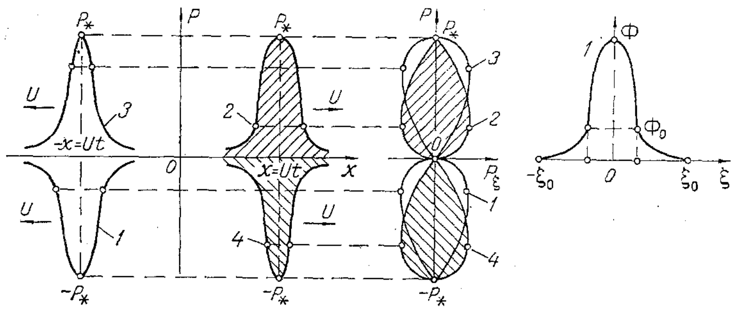

Fig. 3. Characteristic form of the solution (1.7) for waves of the type 1, 2, 3, 4, and their corresponding phase portraits.

By satisfying the conditions P = 0, z = 0, and $P = P_*$, z = 0, taken, respectively, at the outer boundaries of the perturbation wave and at its maximum, we defined $z[P(\xi)]$ in the form

$$z^n = E\varphi(P) \geqslant 0,\ E = -kUP_*/\varkappa, \tag{1.6}$$

$$\varphi = \Phi_1 - \Phi,\ \Phi = Y/Y_*,\ \Phi_1 = (\exp Y - 1)/(\exp Y_* - 1),$$

$$Y = bP,\ Y_* = bP_*,\ b = \varkappa/\mu U;\ S = -kU[1 - Y_*/(\exp Y_* - 1)]/b.$$

For $Y \gtrless 0$ ($Y_* > 0$, $Y_* < 0$, respectively) the functions Φ_1, Φ are nonnegative on the segments $[0; \pm|Y_*|]$. Their difference φ vanishes at the ends of the segments ($P = 0$, $\Phi_1 = \Phi = 0$; $P = P_*$, $\Phi_1 = \Phi = 1$), is negative for $Y > 0$, $\Phi_1 = \Phi_1^+ > 0$, $\varphi = \Phi_1^+ - \Phi = \varphi^+ < 0$, and positive for $Y < 0$, $\Phi_1 = \Phi_1^- > 0$, $\varphi = \Phi_1^- - \Phi = \varphi^- > 0$ (Fig. 1).

Equality of the angles $|\Phi_1^+, Y(0)| = |\Phi_1^-, Y(|Y_*|)$; $|\Phi_1^+, Y(|Y_*|) = |\Phi_1^-, Y(0)|$; $|\varphi_Y^+(|Y_*|) = |\varphi_Y^-(0)|$; $|\varphi_Y^+(0)| = |\varphi_Y^-(|Y_*|)|$ and relating relationships of the form $|Y_m|^+ + |Y_m|^- = |Y_*|$ $\varphi_m^+ + \varphi_m^- = 0$, at points for the maximum of the functions φ^+, φ^- hold.

The condition $z^n > 0$ within the intervals $(0; \pm|Y_*|)$ and $z^n = 0$ at their endpoints $P = 0$ and $P = P_*$ $(P_* \neq 0)$ is satisfied for $\mu > 0$ for the combinations: 1) $U > 0$, P, $P_* < 0$ (on the branch φ^-); 2) $U < 0$, P, $P_* > 0$ (on the branch φ^-), 3) $U > 0$, P, $P_* > 0$ (on φ^+); 4) $U < 0$, P, $P_* < 0$ (on φ^+). For $\mu < 0$ the condition mentioned is not assured for any nonzero, bounded U, P, P_*, and single waves are not formed in the system (1.1) and (1.2) (forces counteracting wave spreading are here replaced by oppositely directed waves).

The phase trajectories of (1.6) with $z = \pm P_\xi$ taken into account possess symmetry with respect to the Y and P axes, and the functions $|\varphi|^+$ $|\varphi|^-$, which vary, respectively, in $[0; +|Y_*|]$ and $[-|Y_*|; 0]$, are governed by the conditions 1), 2), 3), 4) on the segments $[0; \pm|P_*|]$ each (Fig. 2). Here $P_\xi = 0$, $z^n = 0$, $(P_\xi)_P = \pm\infty$, $0 < |(z^n)_P| < \infty$ hold at the point $P = 0$ and $P = P_*$ for all combinations 1), 2), 3), 4).

The solution (1.5) in the form of the divergent integral

$$\int_{P_*}^{P} \frac{dP}{|\varphi(P)|^{1/n}} = \pm |E|^{1/n}\xi;\ 0 \leqslant |P| \leqslant |P_*|, \tag{1.7}$$

written for an arbitrary combination 1, 2, 3, 4 satisfies the condition $P = 0$, $\varphi(0) = 0$, $\xi = \pm\infty$ and $P = P_*$, $\varphi(P_*) = 0$, $\xi = 0$ for each and has the form of single waves.

The characteristic shape of the waves formed by an initial pulse on the axis $x = 0$ is shown in Fig. 3. Here 2 and 3 are a diverging nonsymmetric pair of waves under pulse pumping; 1 and 4 are for pulse selection; 1, 2 and 3, 4 are symmetric pairs of waves under shock action (number of the curves corresponds to the combinations presented in the text; the points $\varphi_m^\pm$ correspond to the inflections of the curves $P(\xi)$ according to (1.6) and (1.7)).

The nonequilibrium filtration velocity is determined from (1.1) and (1.5) as $\mathbf{W} = -m\beta\{\varkappa z^n - \mu U[(z^n)_P - (z^n)_P^0]\}\mathbf{P}_\xi/z = -m\beta\{kUP + S + \mu U(z^n)_P^0\}\mathbf{P}_\xi/z$, and because $S = -\mu U(z^n)_P^0$ from (1.5), $\mathbf{W} = -m\beta kUP\frac{\mathbf{P}_\xi}{z}$. The velocity with zero acceleration ($W_t \sim z$) reaches the maximum at the crest of the wave, changes direction and decreases monotonically to zero at the tail of the waves; from (1.6), for $z \to 0$, $(z^n)_t = Uz(z^n)_P = z\text{const} \to 0$, satisfying the asymptotic taken with $\alpha = 1$. This latter is refined by the boundary conditions of the problem.

In problems without initial conditions, the flow from $(z^n)_P^0 = \text{const}$ can be interpreted as a certain initial (phonon) flow W_S along which there is an impulsive action at $x = 0$ at the time $t = 0$; the magnitude and direction of this flow are selected by starting from the conditions on the external tails of the waves turned to the domain of the initial (unperturbed) state ($\xi = +\infty$ wave 2, $\xi = -\infty$ wave 3, etc.). Here $W_S = m\beta S = -m\beta kU^2[1 - Y_*/(\exp Y_* - 1)]\nu/\lambda$; then $W_S = W_S^+ < 0$ for waves 3 and 4 with $Y_* > 0$, $W_S = W_S^- > 0$ for waves 1 and 2 with $Y_* < 0$, and by directing W_S oppositely to $\mathbf{P}_\xi$ at the external tails of the waves, we have the initial flows $\mathbf{W}_{S3} < 0$ in the domain $x < 0$ for waves 3 and 2, say, $\mathbf{W}_{S2} < 0$ in $x > 0$, and therefore wave 3 is propagated oppositely to the flow $\mathbf{W}_{S3}$ and wave 2 is propagated along the flow $\mathbf{W}_{S2}$ (whereby the asymmetry of waves 2 and 3).

Let us examine the behavior of the system (1.3) as $\mu \to 0$, when the nonequilibrium effects vanish. The coefficient μ in (1.3)-(1.5) is a small parameter in the highest derivative and the passage to $\mu = 0$ results in the appearance of boundary layers in the neighborhoods: I) wave crest $P = P_*$, $z = 0$, and II) tail zones of the waves $P = 0$, $z = 0$. From (1.5) for a boundary layer of the first kind $z^n = -kUP_*(1 - \Phi)/\varkappa > 0$ for 1) $U > 0$; P, $P_* < 0$; 2) $U < 0$; P, $P_* > 0$, and for a boundary layer of the second kind (at the tail of the wave) $z^n = kUP/\varkappa \geqslant 0$; 3) $U > 0$, $P \geqslant 0$ and 4) $U < 0$, $P \leqslant 0$.

Taking account of $z = \pm P_\xi$, the phase trajectories are presented in Fig. 4 for the case mentioned (the dashes show the asymptotic of the branches at a sufficient distance from the boundary layers generating them). Let us present the solution (1.5) for $\mu = 0$ and conditions I and II of the form:

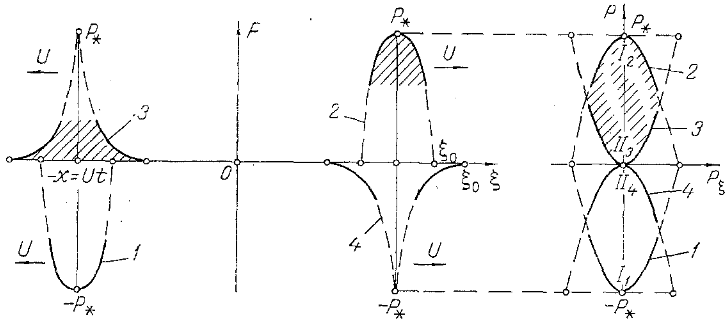

Fig. 4. Characteristic form of the limit waves from (1.8) and (1.9) and their corresponding phase portraits.

$$\Phi = 1 - |r\xi|^{n/(n-1)},\ \Phi_0 \leqslant \Phi \leqslant 1, \tag{1.8}$$

$$\Phi = \begin{cases} (1 - |r\xi|)^{n/(n-1)},\ 0 \leqslant \Phi \leqslant \Phi_0,\ |\xi| \leqslant |\xi_0|, \\ 0,\ |\xi| > |\xi_0|;\ r = |kUP_*^{(1-n)}/\varkappa|^{1/n}(n-1)/n. \end{cases} \tag{1.9}$$

On the outer boundaries $\xi = \xi_0$, $P = 0$, $P_\xi(\xi_0) = 0$ of (1.9), a smooth transition of the solution into the unperturbed domain $P \equiv 0$ is assured for $|\xi| = |\xi_0|$.

Continuation of sections of the phase trajectories $I(\Phi_0 \to 0)$ and $II(\Phi_0 \to 1)$ outside the limits of the corresponding boundary layers yields: for branch I, according to (1.8), for $\xi = \xi_0 = \pm 1/r$, $P = 0$, $P_\xi(\xi_0) = \pm|kUP_*/\varkappa|^{1/n} \neq 0$ and a smooth juncture (1.8) with the unperturbed domain $\Phi \equiv 0$ outside $|\xi| > |\xi_0|$ is not assured; for branch II, according to (1.9), for $\xi = 0$, $P = P_*$, $P_\xi(0) = \pm|kUP_*/\varkappa|^{1/n}$ and the filtration velocity changes direction by a jump at the front.

Let us note that if we pass to the limit $\mu = +0$ for combinations 1, 2 in (1.6), then $\Phi_1 \equiv 1$ $(S \equiv -kUP_*)$ only for $0 < |P| \leqslant |P_*|$ but $P = 0$, $\Phi_1 = 0$ at the point itself; therefore, here $z^n = kUP/\varkappa = 0$ and $\Phi_\xi(\xi_0) = 0$, i.e., we eliminate the discontinuity (1.9) at the tail of the wave. Furthermore, if we pass to the limit $\mu = +0$ for the combinations 3, 4 then $\Phi_1 \equiv 0$ $(S \equiv 0)$ only for $|P| < |P_*|$, but $P = P_*$ $\Phi_1 = 1$ at the point itself; therefore, on the front itself $z^n = -kUP_*(1 - \Phi)/\varkappa = 0$ and $\Phi_\xi(P_*) = 0$, i.e., we also eliminate the discontinuity (1.9) on the wave front. (For generality, if we approach zero from the unstable side $\mu = -0$, then the wave (1.8) of the type 3, 4 will change places with the waves (1.9) of the type 1, 2 and conversely.) Each of the limit waves (1.8), (1.9) satisfies the mass conservation integral relation resulting from (1.2) $|m\beta P_* \int\limits_{-\xi_0}^{\xi_0} \Phi d\xi| = |\lambda P_*^n \int\limits_{-\xi_0}^{\xi_0} (|\Phi_\xi|^n - |\Phi_\xi(\xi_0)|^n)\, d\xi/kU|$, i.e., (1.8), (1.9) can be considered as generalized solutions (1.3)-(1.5) for $\mu = 0$.

The characteristic form of (1.8) and (1.9) is shown in Fig. 4, where the numbering of the curves corresponds to the combinations presented in the text. For small nonequilibrium effects $\delta = |\tau[(z^n)_t - (z^n)_t^0]/z^n| = |\nu U\,(z^n)_P/\lambda z^n - W_S/W_0| < 1$, we have in order of magnitude $W_S \approx W_0$, $O(z) = |P_*/\xi_0|$, $O(z_P) = |1/\xi_0|$, and finally, $\delta = |n/Y_* - 1| < 1$ or $|U| < 2\lambda P_*/\nu n$ for waves with $Y_* > 0$.

To determine the divergent wave velocities and amplitudes, conservation conditions are used for the mass $2M_0$ and the initial momentum $2M$ concentrated in the neighborhood of $x = 0$, and also the relationships connecting the flows $W_S^\pm$ to the wave parameters $m\beta S = W_S^\pm$.

Two dependences (the plus superscript is for the wave $x + Ut$, and the minus for $x - Ut$; the subscript is the wave number

$$2|M_0| = m\rho_0\beta(|P_*^- j^-|_{2,4} + |P_*^+ j^+|_{3,1});\ j^\pm = \int\limits_{-\xi_0}^{\xi_0} \Phi^\pm d\xi,$$

$$2|M| = m\rho_0\beta(J_{2,4}^- + J_{1,3}^+);\ J^\pm = |kU^\pm P_*^\pm j^\pm| \tag{1.10}$$

and the two appropriate coupling equations $W_S^\pm = m\beta S$ determine four parameters of any pair of waves — their amplitudes and velocities. If the initial momentum is of the nature of an im-

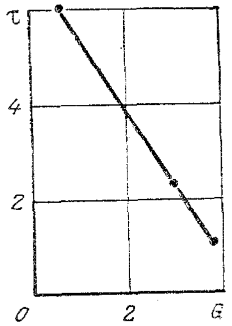

Fig. 5. Dependence of the relaxation time τ (h) on the steady state mass flow rate G (g/min).

pact (rapid pumping with subsequent discharge, or conversely), then the quantity $M_0 = 0$ and we have $j_2^- = -j_1^+$ or $j_3^+ = -j_4^-$ from (1.10), i.e., a diverging symmetric pair of waves soliton–antisoliton of type 1, 2 or 3, 4.

For the limit waves 1, 2 (1.8) we have $|j|_{1,2} = 2n/(2n-1)r_{1,2}$, for the limit waves 3, 4 (1.9) $-|j|_{3,4} = 2(n-1)/(2n-1)r_{3,4}$, and for the diverging pairs of waves 3-2 and 1-4 that each entrain half the momentum of the mass and the motion, we have the estimates

$$|M/M_0| = U_{1,2,3,4} = U,$$
$$|P_*|_{3,4} = [(2n-1)\,(kU/\varkappa)^{1/n}\,|M_0|/2nm\beta\rho_0]^{n/(2n-1)},$$
$$|P_*|_{1,2} = [(2n-1)\,(n-1)\,(kU/\varkappa)^{1/n}\,|M_0|/2n^2m\beta\rho_0]^{n/(2n-1)}.$$

The characteristic relaxation time of the nonequilibrium effects is estimated at the order of magnitude $n\nu/\lambda z = \tau_0$ because of the lack of experimental data, where τ_0 is the relaxation time of a linear ($n = 1$) nonequilibrium system [3] on the order of 2 h, and z is the characteristic pressure drop in the model. Setting $z = 1$ atm/m, we have $n\nu/\lambda = 2$ atm·h/m; therefore, the amplitude is $|P_*| > |Un\nu/2\lambda| = 5$ atm for the velocity $|U| \approx |M/M_0| = 5$ m/h.

The corresponding class of problems of the theory of heat conduction and molecular diffusion is reduced to the results obtained in Sec. 1 by similarity.

2. Thermoconvective Waves. We limit ourselves to the assumptions taken in the theory of heat and mass transfer in porous media for the determination of the energy conservation equation (e.g., [1]):

$$(c\Theta)_t = -(\mathbf{q})_x - c^0\mathbf{W}\Theta_x;\ (c,\ c^0 > 0), \tag{2.1}$$

in which the heat conducting flux $\mathbf{q}$ is determined as $z = |\Theta_x|$:

$$\mathbf{q} = -\lambda^0\{z^n - \tau^0[(z^n)_t - (z^n)_t^0]\}\,\Theta_x/z;\ \tau^0 = \frac{\nu^0}{\lambda^0 z}. \tag{2.2}$$

If W, c, c^0 = const in (2.1), then the wave solutions (2.1), (2.2) reduce to those described in Sec. 1; it is hence sufficient to make the following change in notation in all the relationships of Sec. 1: $\varkappa$ is replaced ($\to$) by λ^0/c; $\mu \to \nu^0/c$; $P(\xi) \to T(\zeta)$; $\xi = x + Ut \to \zeta = x + gt$; $U \to g$, where g is the velocity of the temperature waves, and T is the difference between the running and initial temperatures of the medium $(\partial/\partial t = U\partial/\partial\xi \to g\partial/\partial\zeta;\ \partial/\partial x = \partial/\partial\xi \to \partial/\partial\zeta)$.

If the direction of the convective flux $\mathbf{W}$ agrees with the direction of the divergent temperature waves $\mathbf{g}$ formed by a thermal pulse on the axis of the tunnel, e.g., when pumping fluid into strata, then $k \to 1 - Wc^0/gc$. If the direction of the convective flux is opposite to the direction of the temperature pulses diverging from the tunnel, as, e.g., in withdrawing fluid from a stratum, then $k \to 1 + Wc^0/gc$.

Temperature waves being propagated in the convective flux direction (with heating or cooling) are formed by the combinations: 1) $g > 0$, $W > 0$, T, $T_* < 0$; 2) $g < 0$, $W < 0$, T,

$T_* > 0$; 3) $g > 0$, $W > 0$, T, $T_* > 0$; 4) $g < 0$, $W < 0$, T, $T_* < 0$, and hold for both systems with counteraction ($\nu^o > 0$) for small $|c^oW/cg| < 1$ and with acceleration ($\nu^o < 0$) for large $|c^oW/cg| > 1$.

For temperature waves being propagated opposite to the convective flux, the above-mentioned combinations 1-4 are formed only in systems with counteraction for any $|c^oW/cg|$.

For $\nu^o = 0$ the temperature waves have limit solutions of the type (1.8) and (1.9). The results obtained in Sec. 2 are valid even for an appropriate class of convective diffusion problems.

3. Remarks. 1. We change the initial hypotheses: We consider the filtration flux to be an equilibrium function of an appropriate gradient of the form

$$\mathbf{W} \equiv \mathbf{W}_0 = -\lambda z^n \mathbf{P}_x / z, \tag{3.1}$$

but the fluid density is nonequilibrium. In addition, let the density in (1.2) depend on the change in filtration velocity as

$$\rho = \rho_0 [\beta P + \nu (z^n)_P / m]. \tag{3.2}$$

The first component in (3.2) determines the equilibrium value of the density. Substituting (3.1) and (3.2) into (1.2), we again arrive at the relation (1.5), and later (repeating its solution under the same boundary conditions) at all the subsequent relationships (1.6)-(1.10) describing the soliton waves considered above.

Continuity of the density as well as of the flux $\mathbf{W}$ in the whole ξ-space including at $\xi = 0$ and $\xi = \pm\infty$, where $W = 0$, is assured under the approach mentioned. At the tails of the hydrodynamic waves ($P = 0$, $\xi = \pm\infty$) a nonzero equilibrium fluid density $\rho_S = \rho_0 \nu (z^n)_P / m = -\rho_0 W_S / mU = \rho_0 \nu \beta k U [1 - Y_* / (\exp Y_* - 1)] / \lambda$ appears, where $\rho_S > 0$ for the waves 2, 3 and $\rho_S < 0$ for the waves 1, 4.

Therefore, waves of the kind considered develop in both systems initially mobile and initially at rest with a density different from the equilibrium level. In this respect, the hypotheses about nonequilibrium fluxes and the nonequilibrium density are equivalent; in both cases the system possesses a certain additional energy (with respect to the equilibrium state) before the impulsive action, which is used in soliton motion.

In oil-gas strata, the energetic states mentioned, which are associated with the nonequilibrium of the density, can be produced because of extraction (absorption) of the dissolved, adsorbed gas, thermal action on the stratum, etc.

2. Processing the results of investigations [3] on the filtration of certain non-Newtonian fluids in tubular models with constant pressure drops in the time between the input and output discloses a lower relaxation time close to the inverse dependence $\tau = \nu/\lambda z$ taken for large values of the steady filtration velocity $W_0 = \lambda z^n$ (Fig. 5; here $G = W_0 \rho_0 F$ is the steady-state weight flow rate, and F is the tube section area).

3. The approximation of $\varphi(P)$ by a polynomial $\alpha\Phi^2 + \gamma\Phi^3$, $\alpha = -\gamma = \varphi_m / (1 - \Phi_m)\Phi_m^2$ satisfying $\varphi(\Phi_m) = \varphi_m$, $\varphi(0) = \varphi(1) = 0$, yield the solution (1.7) in the form $\Phi = \operatorname{sech}^2(2\sqrt{|\alpha|}\, r\xi)$ for $n = 2$, where sech is the hyperbolic secant, $\Phi_m = \{\ln[(\exp Y_* - 1)/Y_*]\}/Y_*$, $\varphi_m = -\Phi_m + \{[(\exp Y_* - 1)/Y_*]^{1/Y_*} - 1\}/(\exp Y_* - 1)$. For approximate estimates, $\Phi \approx \{\operatorname{sech}[\pm|\alpha|^{(n-1)/n} |nr\xi/(n-1)|^{n-1}]\}^{n/(n-1)}$; $n > 1$.

NOTATION

$\mathbf{W}$, $\mathbf{q}$ are filtration velocity and heat-conducting flux vectors; P, T, pressure-drop and temperature symbols; λ, λ^o, mobility coefficients of the filtration and heat-conducting fluxes; ν, ν^o, nonequilibrium coefficients of the mass and heat fluxes; c, c^o, volume specific heats of the saturated porous medium and the saturating fluid; ρ, β, porosity, density, and compressibility; n, k, exponents in the transport and similarity coefficient laws; $\mathbf{U}$, $\mathbf{g}$, velocity vectors of the hydrodynamic and thermal waves; P_x, P_t, P_ξ, $\partial/\partial x$, partial derivative symbols; P_*, T_*, stationary P and T values at the wave crest; Θ, Θ_H, running and initial temperatures of the medium; x, t, ξ, y, coordinates; and c, ρ_0, equilibrium specific heat of a porous medium and the fluid density at the initial pressure.

LITERATURE CITED

1. E. Scott, Waves in Nonlinear and Active Media in Application to Electronics [Russian translation], Sovetskoe Radio, Moscow (1977).

2. G. I. Barenblatt, V. M. Entov, and V. K. Ryzhik, Theory of Nonstationary Filtration [in Russian], Nedra, Moscow (1972), pp. 18, 238.
3. P. M. Ogibalov and A. Kh. Mirzadzhanzade, Mechanics of Physical Processes [in Russian], Moscow State Univ. (1976), pp. 18, 68.

STABILITY OF A FILM FLOWING DOWN ALONG AN OSCILLATING SURFACE

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UDC 532.516

The linear approximation for a harmonically oscillating surface is used to obtain the condition of flow stability for a liquid film.

The flow of films in heat and mass transfer devices is nearly always accompanied by wave phenomena at the gas—liquid boundary. The waves considerably affect the transfer processes, and, whenever possible, various adaptations are used which assist the formation of waves or turbulence of the liquid. For example, the vibration of a straight surface can, according to the experimental data [1], lead to an increase of the heat-transfer coefficient by 400%, in comparison with the usual gravitational flow. It is therefore of interest to determine the transition from the waveless regime of the flow to the laminar-wave regime, and then to the turbulent regime, i.e., it is necessary to establish the limits of stability of the particular flow regime in question.

Let us assume that a film of viscous incompressible liquid flows down along a sloped surface which oscillates in its own plane with velocity $V_0 \cos\omega_* \tau$ (Fig. 1). The problem is described by the system of equations

$$\nu \frac{\partial^2 v_1}{\partial x_2^2} + g \sin\gamma = \frac{\partial v_1}{\partial \tau}; \quad \frac{\partial p_\text{d}}{\partial x_2} = \rho g \cos\gamma; \quad \frac{\partial p_\text{d}}{\partial x_3} = 0. \tag{1}$$

In addition, we use the conditions of sticking at the wall, and the absence of tangential stress at the free surface:

$$p_\text{d}(0) = p_\text{atm}; \; v_1|_{x_2=d} = V_0 \cos\omega_* \tau; \; \left.\frac{\partial v_1}{\partial x_2}\right|_{x_2=0} = 0. \tag{2}$$

By solving the system of equations (1) and (2), we determine the unperturbed flow of the layer in the form

$$u_0 = \frac{1}{2}\,\mathrm{Re}\,\mathrm{Fr}^{-1}(1 - y^2)\sin\gamma + \frac{1}{2}\exp(i\omega t)\frac{\mathrm{ch}(1+i)\beta y}{\mathrm{ch}(1+i)\beta} + \frac{1}{2}\exp(-i\omega t)\frac{\mathrm{ch}(1-i)\beta y}{\mathrm{ch}(1-i)\beta};$$

$$p = \frac{y}{\mathrm{Fr}}\cos\gamma + p_a \tag{3}$$

or

$$u_0 = \frac{1}{2}\,\mathrm{Re}\,\mathrm{Fr}^{-1}(1 - y^2)\sin\gamma + A\cos(\omega t - t_\alpha). \tag{4}$$

Here $\mathrm{Re} = V_0 d/\nu$ is the vibrational Reynolds number, $\mathrm{Fr} = \frac{V_0^2}{gd}$, vibrational Froud number; $2\beta^2 = \omega \mathrm{Re}$;

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